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Dynamic multi-period recycling collection routing with uncertain material quality

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Abstract

We consider a problem of collecting and processing waste material. At a production facility, every period, a known amount of inventory is required for production (e.g., paper). Instead of new material, the facility relies on collected and processed waste material (e.g., paper waste). This material is collected from regional waste collection locations. The amount of waste material per location is uncertain, as is the quality of the collected waste, i.e., the resulting inventory when processing the material. If the inventory at the end of a period is insufficient, costly new material has to be bought. Each period, decisions are made about how much waste material to collect from which location and how to route the collection vehicles accordingly. Ideally, inventory is built to hedge against quality uncertainty and to ensure efficient routing operations in future periods. We propose a stochastic lookahead method that samples a set of scenarios and solves a simplified two-stage stochastic program in every period. We show the value of our method for two case studies, one based on real-world data from Sachsen-Anhalt, Germany, and one from the literature with data from the United Kingdom. We further conduct a detailed analysis of our method and the problem characteristics.

Keywords: Routing, Circular Economy, Sequential Decision Process, Stochastic Lookahead

1. Introduction

The circular economy proposes replacing linear sourcing of production materials via global supply chains with recycling of local waste materials (such as paper, plastics, glass, metals, electronic parts, etc.). These materials are collected from local waste suppliers (e.g. waste facilities), cleaned, sorted, and then integrated in the production process. Circular economy projects kick off worldwide and the European Union recently issued a *Circular Economy Action*

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Plan (https://environment.ec.europa.eu/strategy/circular-economy-action-plan_en). In the German state of Sachsen-Anhalt, a large research consortium of Universität Magdeburg, Fraunhofer Institute for Factory Operation and Automation IFF Magdeburg, and Max Planck Institute for Dynamics of Complex Technical Systems Magdeburg work on improving operations in the circular economy in their research cluster *SmartProSys* (https://www.smartprosys.ovgu.de).

There are many reasons for the surge in circular economy projects, not only the high cost of new material and the increasing uncertainty in global supply chains due to political conflicts and natural disasters, but also the commitment to more environmentally sustainable production. Replacing classical linear sourcing of new materials with the use of recycled local materials brings several new challenges in production planning and logistics. In this work, we focus on three of these challenges related to the collection of materials from the suppliers to ensure a steady availability of materials for production: First, in contrast to large shipments of new goods, collecting smaller batches of waste materials from local suppliers requires detailed considerations of routing and the corresponding transportation cost. Second, the available quantities of materials at the local suppliers vary since the quantities depend on the amounts of waste collected. Third, the collected waste materials require cleaning and sorting. As the quality of the waste materials is uncertain, the final quantity that can be used is uncertain as well. These three challenges combined lead to a complex planning problem for companies relying on circular economy sourcing.

The problem can be modeled as a stochastic dynamic multi-period inventory routing problem. There are two stochastic components, first, the available quantities at each supplier in the next period(s) and second, the net quantity of materials that can eventually be used for production. Inventory of net material at the production facility can be built with negligible holding cost, however, the quantity of waste materials that can be processed in every period is limited. In every period, decisions are made about the suppliers to visit for collection, the quantity to collect from each supplier, and the routing of transportation trucks for collections. Besides transportation cost, linear backorder sourcing cost can occur in case the net quantity does not satisfy the material demand at the production facility and new material has to be used instead. The challenge is now to determine collection quantities and routes that are cost efficient, hedge against potential backorder sourcing cost due to insufficient net quantities, and ideally, build some inventory for future periods.

We approach the challenges as follows. First, in every period, we generate a set of multi-period scenarios to capture the two sources of uncertainty. Each scenario contains realizations of waste quantities at the suppliers in the next periods as well as, for each supplier and the current and future periods, the percentages of collected quantities that can be used for production (i.e., the net quantities). Based on the scenarios, a two-stage stochastic program is solved to determine the suppliers to visit and the quantities to collect. Instead of integrating routing decisions explicitly, routing is approximated in the stochastic program. Once the suppliers to visit and the quantities to collect are determined, the detailed routing solution is determined via a routing heuristic from the literature.

We apply our method to a real-world case in Sachsen-Anhalt with real geographical data of production and waste facilities as well as real supply data for paper waste material. We also test our method for the waste collection data provided by Keskin et al. (2023). In a comprehensive computational study, we derive the following managerial insights:

- Anticipatory decision making can reduce cost significantly. Ideally, a method does both intra- and inter-period anticipation in an integrated manner.
- In contrast to myopic procedures, anticipation is able to handle even larger uncertainty in quality loss without noticeable cost increases.
- Expected quality loss is *one* factor for selecting suppliers to collect from. However, supply volumes and routing efficiency are also very important. The importance of the three individual factors differs with respect to the instance setup.
- Having sufficient processing capacity is important. Interestingly, the utilization of the capacity is not leveled, but there are distinct periods of full utilization for building inventory and others without much processed materials.

Our paper makes the following contributions:

- We introduce a new and important problem. To the best of our knowledge, we are the first to address a dynamic routing problem with stochastic yield or quality loss. We present a full mathematical model and embed it in the existing literature via a comprehensive survey.
- We address a real-world case of paper collection in Sachsen-Anhalt, Germany. We further transfer our problem and methodology to existing data from the literature. The created data sets are available online (https://www.ms.ovgu.de/Research.html).
- We suggest a tailored solution method to meet the requirements of the problem by extending the general method of Cuellar-Usaquén et al. (2023) to account for stochastic quality loss and processing capacity.
- We present a comprehensive analysis of method and problem characteristics. We further derive valuable insights with respect to both problem and model.

The paper is outlined as follows. In Section 2, the relevant literature is presented. The problem is defined in Section 3. We present our method in Section 4, and the computational study in Section 5. The paper concludes with a summary and outlook in Section 6.

2. Literature

The literature corresponding to the individual components of our problem is extensive. In the main body of the paper, we focus on the most relevant studies. In Appendix A.1, an overview of other related work is presented.

To our knowledge, the problem presented has not been addressed in the literature, encompassing all its features. It involves the integration of dynamic stochastic multi-period inventory management with routing decisions, particularly dealing with uncertainties regarding both the supply and quality of waste material at suppliers. While related problems have been explored in the literature, they often lack the dynamic aspect over a multi-period decision horizon. Habibi et al. (2017, 2019) address decision-making challenges in third-party reverse logistics, focusing on integrating End-of-Life product collection and disassembly processes. The deterministic variant is solved in Habibi et al. (2017), and the stochastic version, considering uncertainties in waste material supply and quality loss, is addressed in Habibi et al. (2019). The authors propose solutions in the form of Two-Phase Iterative Heuristics and a method based on sample average approximation, respectively. In contrast to prior work, our approach involves addressing inventory management and collection routing decisions on a period-by-period basis, anticipating uncertainty values and decisions for future periods while considering the realization of period-specific uncertainties. In our experiments, we adapt their intra-period approach in our benchmark policy *STS*. We confirm that explicit consideration of quality uncertainties is essential for single-period decision-making; however, for our multi-period setting, a combined intra- and inter-period anticipation is substantially more effective.

Keskin et al. (2023) tackle a multi-period dynamic vehicle routing problem for a waste collection company in the United Kingdom, introducing the concept of "touting" to actively approach convenient customers for orders. The supply of waste material from customers is unknown until the customer is contacted. A rolling horizon and simulation model are used to solve the problem, the authors propose rules to identify promising facilities to call based on the expected supply volumes and how they could be integrated into the routes. Empirical results indicate substantial improvements in routing efficiency compared to the no-touting strategy. Unlike our work, the authors do not perform inventory planning over the periods. Similar to the approach of Keskin et al. (2023), we test several supplier selection rules as benchmark policies. Our results indicate that explicit anticipation of current and future routing decisions and uncertainties is superior to the practically-inspired decision rules.

The consideration of material/product quality from suppliers is explored in other contexts. The emphasis on quality loss in supply chains often revolves around the freshness and perishability of products (Rohmer et al., 2019). Addressing a vehicle routing problem, Stellingwerf et al. (2021) employ a time- and temperature-dependent kinetic model to simulate the degradation of products over time. The quality of products changes after each customer visit, attributed to fluctuations in the vehicle's temperature during transportation. A similar perspective is provided by Alvarez et al. (2022), which tackles a production routing problem specifically concerning perishable goods. In this context, the quality of products is characterized by a decay rate over time. On the other hand, in a manufacturing and disassembly context, Laouini et al. (2023) address the material yield of collected products, seeking to satisfy demand for finished products from the collection of recycled product resources. In contrast to our work, the mentioned studies emphasize that product quality/yield is affected over time and does not have a direct impact on collection routes before reaching the depot, as is the case with perishable products. Additionally, while uncertainty in product quality loss is addressed in manufacturing contexts, routing decisions are not involved, and the uncertainty is not related to the quantity to be replenished in the processing center.

Alvarez et al. (2021) address the stochastic inventory routing problem (SIRP) under uncertainty in both product supply and customer demands. The authors propose a heuristic solution method based on the progressive hedging algorithm, delivering high-quality solutions within reasonable running times for problems with a large number of scenarios. In contrast to our work, the authors contemplate uncertainty in the supply of a single supplier and generate anticipation for a single period going forward. Our proposed methodology explicitly considers future uncertainties and routing for several suppliers.

Methodologically, some works share similarities with our work. Elbek et al. (2015) solve a waste collection problem with uncertainty in the amount available in the containers. The problem is modeled as a two-stage stochastic program and solved using a lookahead algorithm. As in our work, the authors dynamically solve the collection policy for each day of the planning horizon but do not take into account the product quality or the inventory and processing capacity at the depot.

Brinkmann et al. (2019, 2020) focus on dynamic bicycle transportation to maintain optimal inventories at bike share stations. The authors propose a lookahead approach to evaluate inventory decisions, which involves sampling future demand and selecting inventory to minimize unmet demand. The time-influenced lookahead horizon is obtained using the value function approximation (VFA). Notably, this horizon does not extend to future routing decisions. In parallel, our research shares similarities that span both problem formulation and methodological aspects, as we also use future samples to inform inventory decisions. However, our approach explicitly integrates routing and associated cost considerations into the forecasting model.

Finally, Cuellar-Usaquén et al. (2023) solve a dynamic stochastic multi-period problem encompassing purchasing, inventory, and routing decisions for a first-mile operation. The lookahead algorithm proposed samples suppliers' purchase prices as well as supply and demand at the depot. In addition, an adaptive algorithm is introduced to capture the consolidation behavior of suppliers in routing decisions. Similar to our approach, Cuellar-Usaquén et al. (2023) considers an approximation for the routing cost. However, in our case, we approximate the routing cost by parameterizing a general discount factor for all suppliers due to loss characteristics and inventory accumulation over the planning horizon.

Besides the discussed most related research, there is other related research from the fields of recycling collection routing, inventory routing, or uncertain supply and quality loss of materials in manufacturing and supply chains. We discuss this research in detail in Appendix A.1. We further integrate the corresponding papers in the following, summarizing table.

Table 1 presents a summary of the relevant literature to this paper, categorized as follows: *Problem features* indicates whether the problem considers dynamic decisions, stochastic parameters, or multi-period aspects, reflecting the impact of decisions over a time horizon longer than one period; *Decisions* specifies whether the problem involves routing, inventory management decisions, or both; *Source of uncertainty* highlights whether the work considers uncertainty in at least one component of the problem; and *Anticipation* identifies whether the proposed method anticipates the impact of a decision on future costs. For deterministic problems, the Anticipation column is marked as 'n/a,' as anticipation is only applicable in stochastic problems.

3. Problem definition

In this section, we define the problem. We first present a problem description and an illustrative example. Then, we model the problem as a sequential decision process.

3.1. Problem description

We consider a problem of a production facility (now called "depot") that, over a longer time horizon, collects waste material from a set of waste collection facilities (now called "suppliers") and reprocesses it to meet the demand for new product (e.g., paper, plastic, metal, electronic device parts).

We assume a planning horizon with a limited number of periods where the product demand is constant in every period. The product inventory in the initial period is zero at the depot and each supplier has an initial amount of supply available. This amount increases from period to period by an uncertain value following a known, supplier-dependent probability distribution. The exact amount available is only known at the beginning of each period. Further, the quality of the collected supply is unknown and only reveals when processing it at the depot. The quality determines the loss of supply when processing it to new product inventory. This quality can also differ from supplier to supplier, e.g., based on their own collection area (e.g., industrial, residential, commercial, etc.) The quality loss, i.e., the percentage of unusable material also follows a known, supplier-dependent probability distribution. The amount of supply that can be processed at the depot is limited per period. Only processed material can be stored as product inventory at the depot. In case the inventory is not enough to satisfy the demand, the remaining amount is delivered by an expensive backorder supplier (without additional routing cost).

Every period, decisions are made about the suppliers to visit, the amount of supply to collect, and the routing of the homogeneous collection vehicles, starting and ending their tour at the depot. Vehicle tours are restricted by both capacity and working time. We further assume split deliveries are prohibited. The goal is to minimize the expected sum of routing and of backorder cost over all periods. While the first part of the cost is deterministic with respect to a decision, the second is stochastic and realizes when the quality of the supply and the resulting (missing) inventory is revealed.

3.2. Example

In Figure 1, we give an example in order to prepare the modeling of the sequential decision process presented at Section 3.3 and to motivate our solution methodology later. The figure shows a

Category	Author	Problem features			Decisions		Source of uncertainty		Anticipation
		Dynamic	Stochastic	Multi-period	Routing	Inventory	Supply	Quality	
IRP	Moin et al. (2011)			✓	✓	✓			n/a
	Mjirda et al. (2014)			✓	✓	✓			n/a
	Habibi et al. (2017)			✓	✓	✓			n/a
	Chitsaz et al. (2019)			✓	✓	✓			n/a
	Rohmer et al. (2019)			✓	✓	✓			n/a
	Bertazzi et al. (2020)			✓	✓	✓			n/a
	Chitsaz et al. (2020)			✓	✓	✓			n/a
	Alvarez et al. (2022)			✓	✓	✓			n/a
	Chi and He (2023)			✓	✓	✓			n/a
	Jieyu et al. (2024)			✓	✓	✓			n/a
	Nolz et al. (2014)		✓	✓	✓	✓	✓		✓
	Mes et al. (2014)	✓	✓	✓	✓	✓	✓		✓
	Habibi et al. (2019)		✓	✓	✓	✓	✓	✓	✓
	Brinkmann et al. (2019)	✓	✓		✓	✓	✓		✓
	Markov et al. (2020)	✓	✓	✓	✓	✓	✓		✓
	Brinkmann et al. (2020)	✓	✓	✓	✓	✓	✓		✓
	Alvarez et al. (2021)		✓	✓	✓	✓	✓		✓
	Liu et al. (2021)		✓	✓	✓	✓	✓		✓
	Frifita et al. (2022)		✓	✓	✓	✓	✓		
	Hasturk et al. (2024)		✓	✓	✓	✓	✓		✓
RMRP /SCM	Inderfurth and Langella (2006)		✓			✓		✓	✓
	Rickli and Camelio (2014)		✓			✓		✓	
	Keyvanshokooh et al. (2016)		✓			✓	✓	✓	✓
	Üster and Hwang (2017)		✓			✓	✓		✓
	Üster and Memişoğlu (2018)		✓			✓		✓	✓
	Liu and Zhang (2018)		✓	✓		✓		✓	✓
	Memişoğlu and Üster (2021)		✓	✓		✓	✓	✓	✓
	Zhou et al. (2022)		✓	✓		✓		✓	✓
	Li et al. (2023)		✓	✓		✓	✓	✓	✓
	Laouini et al. (2023)		✓	✓		✓		✓	✓
VRP	Kim et al. (2009)				✓				n/a
	De Bruecker et al. (2018)				✓				n/a
	Stellingwerf et al. (2021)				✓				n/a
	Ismail and Loh (2009)		✓		✓		✓		
	Gruler et al. (2017)		✓		✓		✓		
	Cook and Lodree (2017)		✓		✓		✓		✓
	Jammeli et al. (2021)		✓		✓		✓		
	Kyriakidis et al. (2020)		✓		✓		✓		✓
	Marković et al. (2020)		✓		✓		✓		
	Sasha Dong et al. (2022)		✓		✓		✓		✓
MP-R	Bogh et al. (2014)			✓	✓				n/a
	Archetti et al. (2015)			✓	✓				n/a
	Larrain et al. (2019)			✓	✓				n/a
	Wen et al. (2010)	✓	✓	✓	✓		✓		✓
	Albareda-Sambola et al. (2014)	✓	✓	✓	✓		✓		
	Cordeau et al. (2015)	✓	✓	✓	✓		✓		✓
	Elbek et al. (2015)	✓	✓	✓	✓		✓		✓
	Subramanyam et al. (2021)	✓	✓	✓	✓		✓		✓
	Keskin et al. (2023)	✓	✓	✓	✓		✓		✓
	Cuellar-Usaquén et al. (2023)	✓	✓	✓	✓	✓	✓		✓
	Our work	✓	✓	✓	✓	✓	✓	✓	✓

Table 1: Literature classification, highlighting similarities and differences of our proposed problem and the literature.

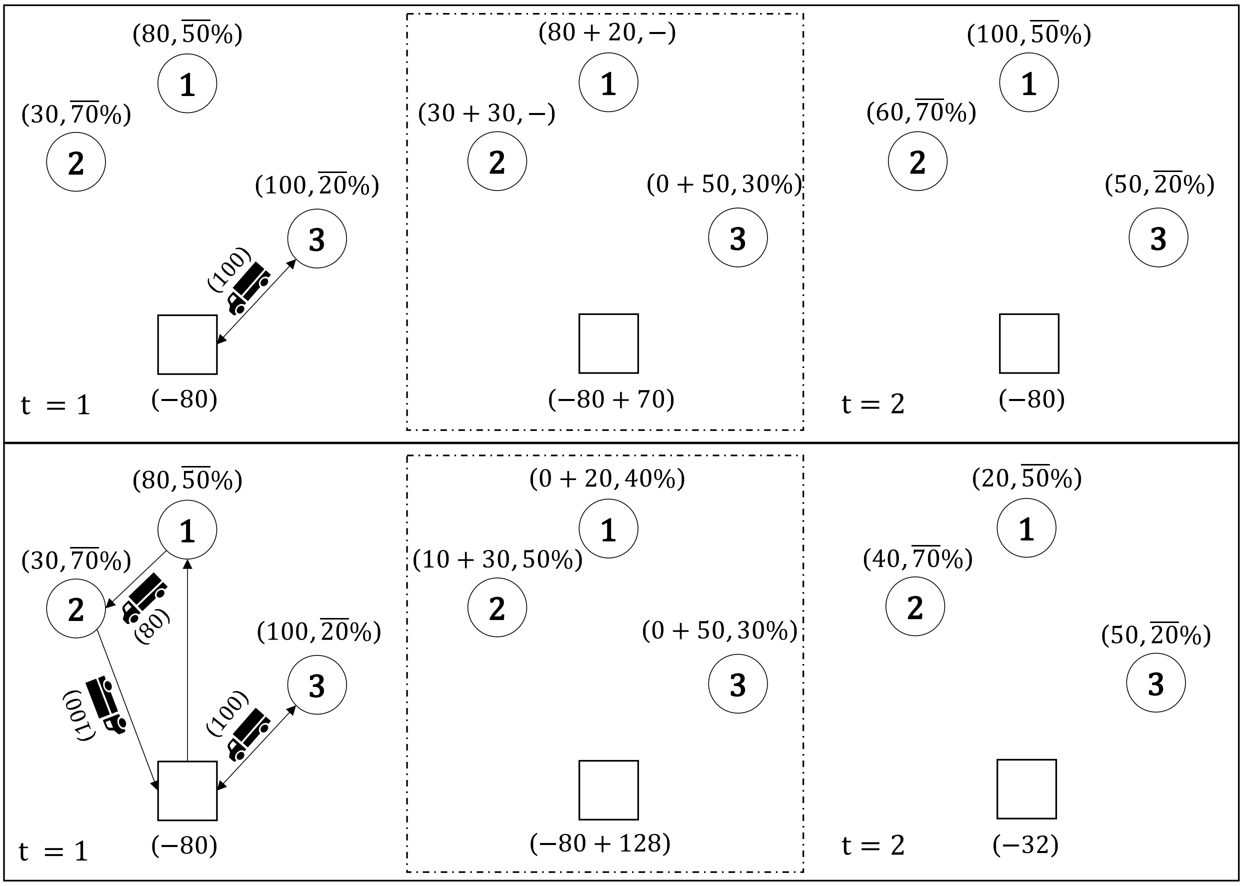


Figure 1: Example of a state, two possible decisions and their corresponding post-decision states, and next states.

state at period $t = 1$ with two potential decisions on the left, the revelation of stochastic information in the center, and new states in $t = 2$ on the right. In the small example, we assume a collection network comprising three suppliers (circles) and one depot (square). The vehicle capacity is set to 100, and the processing capacity is set to 200. For the presentation, we omit the vehicles' travel time limit and travel time details.

We start with describing the state and decisions on the left-hand side of the figure. The first decision is shown top left, the second bottom left. The state information comprises the available inventory minus the demand at the depot. The value -80 indicates a net demand of 80 units in this period. The information for each supplier is twofold and shown next to the supplier's location, e.g., $(80, \bar{50}\%)$ for supplier 1. The first value indicates the available supply. The second value reflects the expected quality loss for each supplier. This value is supplier-specific but stays constant over the periods. For example, supplier 1 has 80 units supply in stock and the expected loss is 50%, i.e., collecting all 80 units will lead to an expected inventory increase of 40 at the depot.

We now consider the two decisions and their consequences in detail. The first potential decision is presented at the top of the Figure 1. This decision seeks to collect what is needed to satisfy demand as efficiently as possible neglecting potential loss uncertainty. Only supplier 3 is visited in a pendulum tour and all supply is transported to the depot. This is indicated by the value 100 next to the vehicle. The top center of the figure now shows the realization of the stochastic information: the realized loss and the realized increase in supply for the next period. As suppliers 1 and 2 are not visited, the loss is not observed, but only the increase is shown, 20 units for supplier 1 and 30 units for supplier 2. For supplier 3, the realized loss is 30%, higher than expected. The increase in supply is 50. The higher than expected loss leads to an inventory increase of only 70. Given the demand of 80, ten units need to be purchased by the (costly) backorder option to reduce the net inventory to zero. The next state is then shown on the right. The inventory value is -80 and the supply values are updated according to the realized increase.

Another potential decision is presented at the bottom of Figure 1. This decision invests routing cost to increase expected inventory at the depot and to avoid orders from the backup option. Two vehicles are employed. The first one collects all supply from supplier 3 similar to the first decision. The second vehicle first visits supplier 1 and collects all 80 available supply units, indicated by the value (80) next to the vehicle when leaving supplier 1. Then, the vehicle visits supplier 2 and collects additional 20 units, returning full back to the depot. As 200 units are collected overall, the production capacity limit is not exceeded. The bottom center shows the realization of stochastic information, similar to before, but now also for the loss for suppliers 1 (40%) and 2 (50%). This leads to an inventory increase of $48 + 10 + 70 = 128$ and $128 - 80 = 48$ inventory units remaining at the end of the period. In the new state on the right, the net inventory is therefore -32.

3.3. Sequential decision process

The problem at hand is a stochastic and dynamic decision problem. It is stochastic because the amount of waste material available at the suppliers known at the beginning of each period and the amount of supply that can be used to generate product inventory is revealed when it arrives at the depot. It is dynamic because a sequence of decisions must be made, one decision per period. In addition, current decisions change the inventory volumes of future periods, which influences future decisions.

A dynamic stochastic decision problem can be modeled as a sequential decision process (Powell, 2021), modeling the problem as a sequence of states. In each state, a decision is made and the cost is observed. Next, stochastic information is revealed (resulting in further cost) and a transition leads to the next state. In the following, we define the states, the decisions, the cost functions, the stochastic information and the transition function of our problem. First, we introduce the global notation. For an overview of all the notation used, we refer to Appendix A.2.

3.3.1. Global notation

We denote the set of suppliers as $m \in M$. We define the collection network as a complete, directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$. Let $\mathcal{V} := M \cup \{0\}$ be the set of vertices, where 0 represents the depot. Let $\mathcal{A}$ be the set of arcs where $\mathcal{A} = \{(i,j) : i,j \in V | i \neq j\}$. For the sub-tour elimination constraints, given a set $\mathcal{U} \subset \mathcal{V}$, we define $\delta^+(\mathcal{U})$ as the set of arcs (i,j) with $i \in \mathcal{U}$ and $j \in \mathcal{V} \setminus \mathcal{U}$, and $\delta^-(\mathcal{U})$ as the set of arcs (i,j) with $j \in \mathcal{U}$ and $i \in \mathcal{V} \setminus \mathcal{U}$.

The periods are denoted as $t \in T$ with $T = \{1, 2, \ldots, |T|\}$. Each period, the demand is the same, denoted as d. For each supplier, the increase in supply per period and the quality loss follow known probability distributions with mean and standard deviation (μ_m^r, σ_m^r) for supply increase value and $(\mu_m^\phi, \sigma_m^\phi)$ for quality loss percentage.

We assume a sufficiently large available set of vehicles F (e.g., $|F| = |M|$). Vehicles have a maximum load capacity Q, and a maximum working time per vehicle and period $l^{\max}$. The travel time between i and j for $(i,j) \in \mathcal{V}$ is denoted by τ_{ij}, and for every time unit traveled there is a cost of c. The service time to load the waste material on the vehicles is same for all suppliers. We include the service times in the travel times τ_{ij}, leading to asymmetric travel times from/to the depot. At the end of the collection, when the vehicles arrive at the depot, the sum of the quantities collected cannot exceed the processing capacity defined as a factor of the demand, $\beta \times d$ with β being the processing capacity scaling parameter. The backorder cost per unit is defined as f.

3.3.2. State

A decision is made in every period $t \in T$. The state comprises all information available to make a decision. We denote the state in period $t \in T$ as S_t. For our problem, the state S_t consists of two components, one related to the depot and one related to the suppliers: The first is the net inventory at the depot in period t, denoted by I_t. The second is the amount of supply available at supplier $m \in M$ at period t, denoted by q_{mt}.

We note that the net inventory already captures the demand for the day and therefore can be negative. State S_t can be summarized as $S_t = (I_t, q_t)$, where I_t is a scalar and q_t is a $|M|$-dimensional vector. In the beginning of the process, there is no inventory available at the depot, $I_0 = -d$. The initial amount of supply of supplier $m \in M$ is denoted q_{m0}.

3.3.3. Decision

We denote a decision at period $t \in T$ as a_t. A decision $a_t = (z_t, x_t)$ has two components that reflect collection and routing parts. The collection part is modeled via decision matrix $z_t = (z_{mvt})_{m \in M, v \in F}$. It determines the amount of waste material to collect from each supplier m by each vehicle v. The second part of the decision is the definition of collection routes, modeled via $x_t = (x_{ijvt})_{i,j \in M, i \neq j, v \in F}$. The variable $x_{ijvt} \in \{0,1\}$ indicates if the arc from supplier i to supplier j is activated in the route of vehicle v at period t. In the following, we summarize the decision space using a mixed-integer formulation. For the formulation, we use the auxiliary variable e_{mft} which takes the value of one if supplier $m \in M$ is visited by vehicle $v \in F$ at period t and 0 otherwise.

A decision $a_t = (z_t, x_t)$ at period $t \in T$ is feasible if the constraints (1)-(11) are satisfied. Eq. (1) ensures that the processing capacity $\beta \times d$ in the depot is not exceeded. Eq. (2) ensures that the collection should not exceed the available supply of waste material of the suppliers and that a vehicle can only collect supply if a supplier is visited. Eq. (3) ensures that the vehicle capacity Q is not exceeded if one or more suppliers are visited. Eq. (4) and Eq. (5) are non-split visit constraints that ensure that each supplier is visited by at most one vehicle and that no more than the vehicle's capacity is collected. Eq. (6) imposes that, for each visited supplier, exactly one arc must enter and leave the relative node. Eq. (7) and Eq. (8) are the sub-tour elimination constraints and maximum travel time constrains (Manerba and Mansini, 2016). Finally, Eqs. (9)-(11) define the domain of the variables.

$$\sum_{v \in F} \sum_{m \in M} z_{mvt} \leq \beta d \tag{1}$$

$$z_{mvt} \leq q_{mt} e_{mvt}, \qquad \forall\, m \in M, \forall\, v \in F \tag{2}$$

$$\sum_{m \in M} z_{mvt} \leq Q, \qquad \forall\, v \in F \tag{3}$$

$$\sum_{v \in F} e_{mvt} \leq 1, \qquad \forall\, m \in M \tag{4}$$

$$z_{mvt} \leq Q, \qquad \forall\, m \in M, \forall\, v \in F \tag{5}$$

$$\sum_{(i,j) \in \delta^-(\{b\})} x_{ijvt} = \sum_{(i,j) \in \delta^+(\{b\})} x_{ijvt} = e_{bvt}, \qquad \forall\, b \in M, \forall\, v \in F \tag{6}$$

$$\sum_{(i,j) \in \delta^+(\mathcal{U})} x_{ijvt} \geq e_{bvt}, \qquad \forall \mathcal{U} \subseteq M, \forall\, b \in \mathcal{U}, \forall\, v \in F \tag{7}$$

$$\sum_{(i,j) \in \mathcal{A}} \tau_{ij} x_{ijvt} \leq l^{max}, \qquad \forall\, v \in F \tag{8}$$

$$z_{mvt} \geq 0, \qquad \forall\, m \in M, \forall\, v \in F \tag{9}$$

$$e_{mvt} \in \{0, 1\}, \qquad \forall\, m \in M, \forall\, v \in F \tag{10}$$

$$x_{ijvt} \in \{0, 1\}, \qquad \forall\, (i, j) \in \mathcal{A}, \forall\, v \in F \tag{11}$$

The routing cost associated with decision a_t in state S_t is determined by multiplying the duration of each route by the cost per unit of time. The routing cost $C^r(S_t, a_t)$ can be formally defined as:

$$C^r(S_t, a_t) = c \cdot \sum_{v \in F} \sum_{(i,j) \in \mathcal{A}} \tau_{ij} x_{ijvt}. \tag{12}$$

3.3.4. Stochastic information and transition function

After a decision a_t is taken in state S_t, the state is transferred to post-decision state $S_t^a = (I_t, q_t^a, z_t)$. The collection decision, z_t, induces changes in the amount of supply available at the suppliers. The final inventory of supply at the suppliers at period t is the difference between the amount of supply on hand and the amount collected:

$$q^a_{mt} = q_{mt} - \sum_{v \in F} z_{mvt},\ \forall\, m \in M. \tag{13}$$

The exogenous information $\omega_{t+1} = (\hat{z}^{\omega}_{t+1}, r^{\omega}_{t+1})$ reveals the inventory that can be generated from the supply collected and the additional supply available at the suppliers. The realization of generated inventory, $\hat{z}^{\omega}_{t+1}$, impacts both the satisfaction of demand and the final inventory at the depot. We define I^{ω}_{t+1} as the difference between the revealed amount of material that can be used and the net inventory level at period t:

$$I^{\omega}_{t+1} = I_t + \hat{z}^{\omega}_{t+1}. \tag{14}$$

In the case that the inventory at the depot is not enough to meet the demand in period t, a backorder cost is incurred for the purchase of material to meet the demand. The realized backorder cost is defined as:

$$C^e(S_t, a_t, \omega_{t+1}) = f \cdot \max(0, -I^{\omega}_{t+1}). \tag{15}$$

After applying the transition function $\mathcal{T}(S^a_t, \omega_{t+1})$, a new state $S_{t+1} = (I_{t+1}, q_{t+1})$ is reached. Firstly, the net inventory I_{t+1} is updated, taking into account the realization of the final inventory level of the new product at the depot and the demand of period $t+1$. Secondly, the supply of waste material available q_{mt+1} is updated based on the final inventory at each supplier and the newly generated amount. We define this update as follows:

$$I_{t+1} = \max(0, I^{\omega}_{t+1}) - d \tag{16}$$

$$q_{mt+1} = q^a_{mt} + r^{\omega}_{mt+1}, \qquad \forall\, m \in M. \tag{17}$$

3.3.5. Policy

A solution for a sequential decision process is a policy π. A policy assigns a decision $a_t = A^{\pi}(S_t)$ to every state S_t. The overall set of policies is defined as Π. An optimal solution $\pi^* \in \Pi$ minimizes the expected cost that is composed of the routing cost and the backorder cost:

$$\pi^* = \underset{\pi \in \Pi}{\operatorname{argmin}}\, \mathbb{E}\left[\sum_{t \in T} (C(S_t, A^{\pi}(S_t)) + C^e(S_t, A^{\pi}(S_t)) | S_0)\right], \tag{18}$$

starting from state S_0. Term $C^e(S_t, A^{\pi}(S_t))$ indicates the expected backorder cost.

4. Method

Even the small example in Section 3 already highlights the challenges in decision making for this problem. First, uncertainty does not only manifest in the supply per period, but also in the loss, i.e., the percentage of collected material that cannot be used. Second, the decision in a state comprises a complex inventory routing problem. Thus, solution methodology must account for the

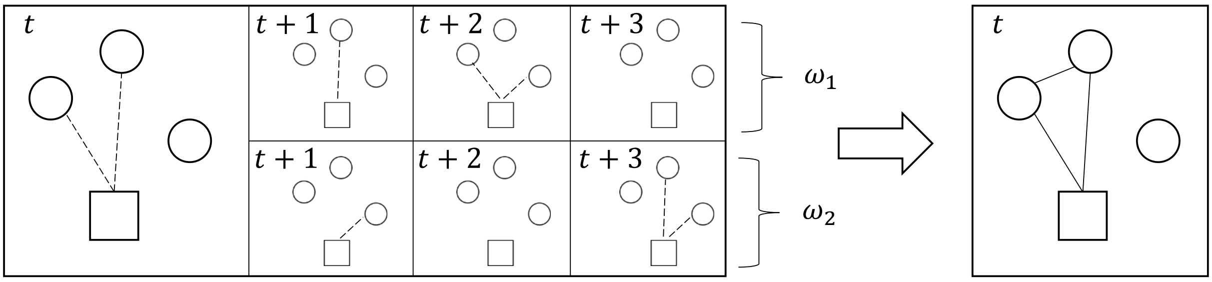


Figure 2: Illustration of the *STM*-policy with simplified two-stage stochastic program on the left and resulting routing decision on the right.

two sources of uncertainty within the period and for the next periods and must be able to tackle the complex decision space thoroughly.

To this end, we propose a stochastic lookahead method adapted from Cuellar-Usaquén et al. (2023), denoted *ST*ochastic lookahead over *M*ultiple periods (*STM*). In every state, STM solves a stochastic lookahead model based on a set of sampled multi-period scenarios. Each scenario comprises the realization of loss in the current period and the realizations of supply and loss in future periods. Then, a decision is taken that minimizes the average cost while considering all possible scenarios. Mathematically, this can be modeled as a two-stage stochastic program. Solving the program is computational intractable, since routing decisions in the current and future periods must be considered. Instead, we assume direct trips but approximate routing consolidation via a discount parameter. After the collection decisions are determined, the detailed routing is done via a heuristic. The concept is illustrated in Figure 2 for the state previously introduced in the example. Here, the process is at time t and we assume two scenarios (ω_1, ω_2), and a horizon of three periods. In the first step, shown on the left, *STM* solves a two-stage stochastic program with one joint decision for time t and individual decisions in the scenarios for times $t+1$, $t+2$, $t+3$. All decisions assume direct trips and are evaluated via discounted direct trip cost. Then, in a second step, shown on the right, the decision at time t is transferred to a routing decision. Cuellar-Usaquén et al. (2023) have shown that this procedure works very well for routing problems with a limited number of stops per vehicle due to capacity constraints, as given for our problem. In the following, we first present the stochastic lookahead model and then the routing heuristic.

4.1. The stochastic lookahead model

In each state S_t during period t, the stochastic lookahead model solves a two-stage stochastic program to determine the amount of supply to collect, inventory levels, and the suppliers to visit from the depot. This lookahead model solves the problem over a shorter planning horizon by combining an approximation of future information with an approximation of future decisions. Therefore, in the lookahead model, a forward period horizon T' is composed for the subsequent periods: $t, t+1, \ldots, t+h$. Notably, decisions related to $t' > t$ are employed to evaluate the quality of decisions to be made in period t. For each period $t' \in T'$, a set of scenarios, i.e., sample paths $\omega \in \Omega$, is constructed. Each sample path determines the realized amount of supply ($r_{mt'\omega}$) and the quality loss percentage ($\phi_{mt'\omega}$) for every period $t' \in T'$ and every supplier $m \in M$.

The two-stage stochastic program is presented in Eqs. (19)-(30). Derived from the model defined in Section 3.3, this stochastic program models multiple scenarios and time periods. The stochastic program simplifies the routing decision by assuming discounted direct trips. To this end, it introduces two additional parameters, $\hat{\tau}_m$ and γ. The parameter $\hat{\tau}_m$ is the direct trip cost from the depot to every supplier m if we collect something from the supplier, with $\hat{\tau}_m := \tau_{0m} + \tau_{m0}, \forall m \in M$. The parameter γ serves as a discount parameter, estimating the routing cost, as direct trips may overestimate the actual routing cost. It takes values in the range $[0, 1]$. A weight close to zero indicates consolidation potential and a weight close to one indicates that the supplier is usually visited by direct travel. The parameter is tuned via enumeration.

The objective function (19) minimizes the expected total cost of collection routing and backorder cost. Eq. (20) guarantees the flow of inventory and ensures demand satisfaction at the depot, considering potential future values of the quality loss percentage. Eq. (21) ensures the flow of inventory and new supply at suppliers. Eq. (22) and Eq. (23) ensure not exceeding the capacity of the vehicles, along with the constraints of no split visits and not exceeding the processing capacity of the depot. Non-anticipativity constraints, represented by Eq. (24) and Eq. (25), ensure that the first-period decisions on the horizon T', corresponding to state S_t, remain consistent across sample paths. Finally, Eqs. (26)-(30) define the variable domains.

Compared with the model presented in Section 3.3, the two-stage stochastic program does not take into account the vehicle-specific routing decision, x_{ijvt} being replaced by supplier selection decisions $e'_{mt'\omega}$. Additionally, a backorder amount variable $y_{t'\omega}$ is added. The constraints (1)-(5) are adjusted to constraints (21)-(23); and the constraints (6)-(8) are removed. For the instances considered in this paper, the stochastic program can be solved using a standard solver setting a maximum gap of 10%. Let $\bar{e}$ and $\bar{z}$ represent the computational results for the decision variables e' and z'. The values of $\bar{e}$ and $\bar{z}$ for period t serve as input information for the routing heuristic.

$$\min \sum_{\omega \in \Omega} \frac{1}{|\Omega|} \left(\sum_{t' \in T'} c \sum_{m \in M} \gamma \hat{\tau}_m e'_{mt'\omega} + f \cdot y_{t'\omega} \right) \tag{19}$$

s.t.

$$I'_{t'\omega} = I'_{t'-1\omega} + \sum_{m \in M} z'_{mt'\omega}(1 - \phi_{mt'\omega}) - d + y_{t'\omega}, \qquad \forall t' \in T', \forall \omega \in \Omega \tag{20}$$

$$k_{mt'\omega} = k_{mt'-1\omega} + r_{mt'\omega} - z'_{mt'\omega}, \qquad \forall m \in M, \forall t' \in T', \forall \omega \in \Omega \tag{21}$$

$$z'_{mt'\omega} \leq Q e'_{mt'\omega}, \qquad \forall m \in M, \forall t' \in T', \forall \omega \in \Omega \tag{22}$$

$$\sum_{m \in M} z'_{mt'\omega} \leq \beta d, \qquad \forall t' \in T', \forall \omega \in \Omega \tag{23}$$

$$z'_{mt\omega} = \sum_{\omega' \in \Omega} \frac{z'_{mt\omega'}}{|\Omega|}, \qquad \forall m \in M, \forall \omega \in \Omega \tag{24}$$

$$e'_{mt\omega} = \sum_{\omega' \in \Omega} \frac{e'_{mt\omega'}}{|\Omega|}, \qquad \forall m \in M, \forall \omega \in \Omega \tag{25}$$

$$e'_{mt'\omega} \in \{0, 1\}, \qquad \forall m \in M, \forall t' \in T', \forall \omega \in \Omega \tag{26}$$

$$z'_{mt'\omega} \geq 0, \qquad \forall m \in M, \forall t' \in T', \forall \omega \in \Omega \tag{27}$$

$$I'_{t'\omega} \geq 0, \qquad \forall t' \in T', \forall \omega \in \Omega \tag{28}$$

$$k_{mt'\omega} \geq 0, \qquad \forall m \in M, \forall t' \in T', \forall \omega \in \Omega \tag{29}$$

$$y_{t'\omega} \geq 0, \qquad \forall t' \in T', \forall \omega \in \Omega \tag{30}$$

4.2. Routing heuristic

After solving the two-stage stochastic program, which returns the values of the supply collected ($\bar{z}$) and supplier selection ($\bar{e}$) at a state S_t, the routing heuristic proceeds to determine the routing decision, x_{ijvt}, by solving a Distance Constrained Capacitated Vehicle Routing Problem (DCVRP). The conceptual process is outlined below, with further details available in Cuellar-Usaquén et al. (2023). First, a giant tour is created using a nearest neighbor algorithm, based on the selected suppliers, following the approach outlined in Cuellar-Usaquén et al. (2021). Subsequently, an augmented graph is constructed, using the giant tour and the supply collected ($\bar{z}$). The split procedure from Prins (2004) is employed to extract the pool of routes from the augmented graph, respecting vehicle capacities and maximum travel time constraints. The construction of the augmented graph follows a Directed Acyclic Graph (DAG) structure. Subsequently, a single-source shortest path problem is solved to find the set of routes that minimizes travel time (Cormen et al., 2022). This involves using a topological ordering of vertices, resulting in a complexity of $O(|A|)$. The resulting routes are then implemented in the decision variable x_{ijvt}, where each route corresponds to a vehicle assignment.

4.3. Implementation details

Based on preliminary experiments, we set the lookahead horizon to three periods, $h = 3$, i.e., the stochastic program considers four periods total. We set the number of scenarios to $|\Omega| = 10$. The

γ-value is fixed to 0.6. For all the experiments, a computer with an 12th Gen Intel(R) Core(TM) i7-12800HX 2.00 GHz was used with Windows 11 and 64 GB RAM. All implementation are coded in Python 3.9, and Gurobi 9.1.1 is used as optimizer. Decisions are usually obtained within a few seconds for all policies.

5. Computational study

In this section, we present our computational study. We first describe the test instances and the benchmark policies. We then analyze the value of single-period and multi-period anticipation of our method. Finally, we investigate the decision making of our method in detail.

5.1. Instances

We present two main instance settings for geography, supply, and demand. One setting is based on the recycling collection data from the German state of Sachsen-Anhalt, the second one is adapted from the United Kingdom (UK) waste collection data presented in Keskin et al. (2023). Amongst others, the instances differ in the number of suppliers and their geographical distribution.

- **Sachsen-Anhalt:** The data comprises locations and supply data of nine paper collection facilities and a paper processing facility spread over the entirety of Sachsen-Anhalt. The travel distances between the locations are calculated via Google Maps on free roads and multiplied by a factor of 1.3 to mimic potential traffic. For each supplier, we have access of the monthly supply data in tons from January 2021 to December 2022. Based on the data, we calculate the expected supply per supplier and per period as the average weekly supply. We assume the supply follows a normal distribution and fit mean and standard deviation to the data accordingly. The expected values range in the interval $\mu \in [0.3, 7.2]$ tons and the standard deviation in the interval $\sigma \in [0.07, 0.9]$.

- **UK-instances from Keskin et al. (2023):** The data comprises travel times and supply data in liters from a waste collection company operating in a smaller region of the United Kingdom. The travel time is obtained using the coordinates of the customers (suppliers). The supply distribution is calculated from a real-world dataset that covers three months of waste collections for two drivers operating from one depot. For these instances, we focused on the customers with a higher waste volume. To this end, we sorted the suppliers by expected value in descending order and selected the first 30 customers. The supply follows a normal distribution with μ ranging in interval $[40.23, 278.64]$ liters for the different suppliers and the standard deviation $\sigma \in [50.04, 512.96]$. Even though negative values are highly unlikely, we truncate the supply distributions at 0 liters (and $2 \cdot \mu$ liters to ensure symmetry).

For both data sets and our main experiments, we assume a time horizon of 20 periods (e.g., reflecting weekly collections). We set the demand per period to 50% percent of the expected supply per period. We further set the processing capacity as two times the demand, and there is no initial inventory of new product at the depot. In practice, the average quality loss for paper is around 20%(AF&PA, 2024). Consequently, the expected loss per supplier is sampled uniformly from a range 0.1 to 0.3. Then, the loss distribution per supplier follow a normal distribution with a coefficient of variation of 0.1. Each supplier has an initial inventory of supply sampled from a normal distribution with a mean of half a vehicle capacity $Q/2$ and a coefficient of variation of 0.1. Based on discussions with domain experts, the vehicle capacity is set to 10 tons for the Sachsen-Anhalt case and 1000 liters for UK-instances. Vehicle travel cost is set to 1.5 EUR per minute (In Appendix A.4, we show that the relative performance of our method is rather invariant to vehicle capacity and cost, except for cases where the vehicle capacity is too small). We fixed a service time of 30 minutes for loading the waste into the vehicles, a maximum working time of 480 minutes, and a vehicle speed of 60 km/hour. The backorder costs are set to 1000 EUR/ton for the Sachsen-Anhalt case (Pimster, 2022), and, equivalently, 10 EUR/liter for UK-instances (In Appendix A.5, we show that the results are relatively stable regardless the backorder cost). We generate 30 instance realizations for each setting to evaluate the average performance of the policies. The tuning of the policies is done on 30 different instance realizations.

5.2. Benchmark policies

In our problem, we propose a method, *STM*, that anticipates loss and supply uncertainty within the period and over the periods and also allows for a consideration of inventory and routing decisions. To analyze the components of our policy, we create two sets of policies. The first set focuses on anticipation within the period (*intra-period*), the second on *multi-period* anticipation.

Intra-Period:. We create five policies for intra-period anticipation, all only considering the current (single) period: one myopic policy, two simple practical strategies, and a single-period variant of our STM-method:

- *Myopic*: This policy aims on minimizing the immediate cost, assuming deterministic information. It applies *STM* with only one scenario (supply realized and expected loss values) and a lookahead horizon of zero.
- *Quality*: This policy aims on predominantly collecting from high-quality suppliers. To this end, the suppliers are sorted by expected loss and selection continues until the cumulative expected value satisfies the demand. In contrast to *Myopic*, this policy does not consider routing in selection of suppliers. Once the selection is complete, the routing is determined similar to *STM*.
- *Volume*: This policy is similar to *Quality* but aims on predominantly collecting from suppliers with high supply volumes. To this end, the suppliers are sorted by current supply volume.
- *Distance*: This policy is similar to *Quality* but aims on predominantly collecting from suppliers close to the depot. To this end, the suppliers are sorted by distance.

- *STS*: This policy is like *STM*, but only looks at the current (single) period. Thus, it anticipates loss uncertainty in the period but not future periods. This is implemented by applying *STM* but with lookahead horizon of zero.

Multi-Period:. We create five policies for multi-period anticipation, each extending one of the five single-period policies:

- *Myopic+M*: This policy assumes deterministic loss and supply values in the current period. However, if possible, it collects a certain percentage of expected supply more than needed in every period. This builds inventory for future periods. Doing so, it also implicitly hedges against loss uncertainty. In case there is sufficient inventory in a period to satisfy all demand, no vehicles are dispatched by this policy. The best percentages for this policy and the following four policies are determined individually via enumeration.
- *Quality+M*: This policy performs similar to *Quality*, but also builds an inventory by collecting a percentage more than needed as done by *Myopic+M*.
- *Volume+M*: This policy performs similar to *Volume*, but also builds an inventory by collecting a percentage more than needed as done by *Myopic+M*.
- *Distance+M*: This policy performs similar to *Distance*, but also builds an inventory by collecting a percentage more than needed as done by *Myopic+M*.
- *STS+M*: This policy performs similar to *STS*, but also builds an inventory by collecting a percentage more than needed as done by *Myopic+M*.

5.3. The value of anticipation

In the following, we compare the policies and show the value of intra-period anticipation as well as combined intra- and inter-period anticipation.

Intra-Period anticipation. In this section, we analyze the intra-period policies. We first compare their objective values and then show the difference in decision making.

Figure 3 shows the performance of the intra-period benchmark policies for the Sachsen-Anhalt case and the UK-instances from Keskin et al. (2023). The policies are depicted on the y-axis. The x-axis presents the average objective value for each policy. For both settings, we observe very similar behavior of the policies. The smaller values for the UK-instances results from the higher number of available suppliers and comparably smaller travel times. For both settings the best performance is achieved by *STS*, followed by *Myopic*. The three policies *Quality*, *Volume*, and *Distance* all perform worse. The particularly poor performance of *Distance* in the UK-instances results from the comparably short distances. Hence, focusing on distance is less important compared to the Sachsen-Anhalt case.

The poor performance of all three policies compared to *STS* and *Myopic* indicates that the consideration of routing in decision making plays a significant role when searching for effective decisions. We further observe that the policy that explicitly considers uncertainty across scenarios,

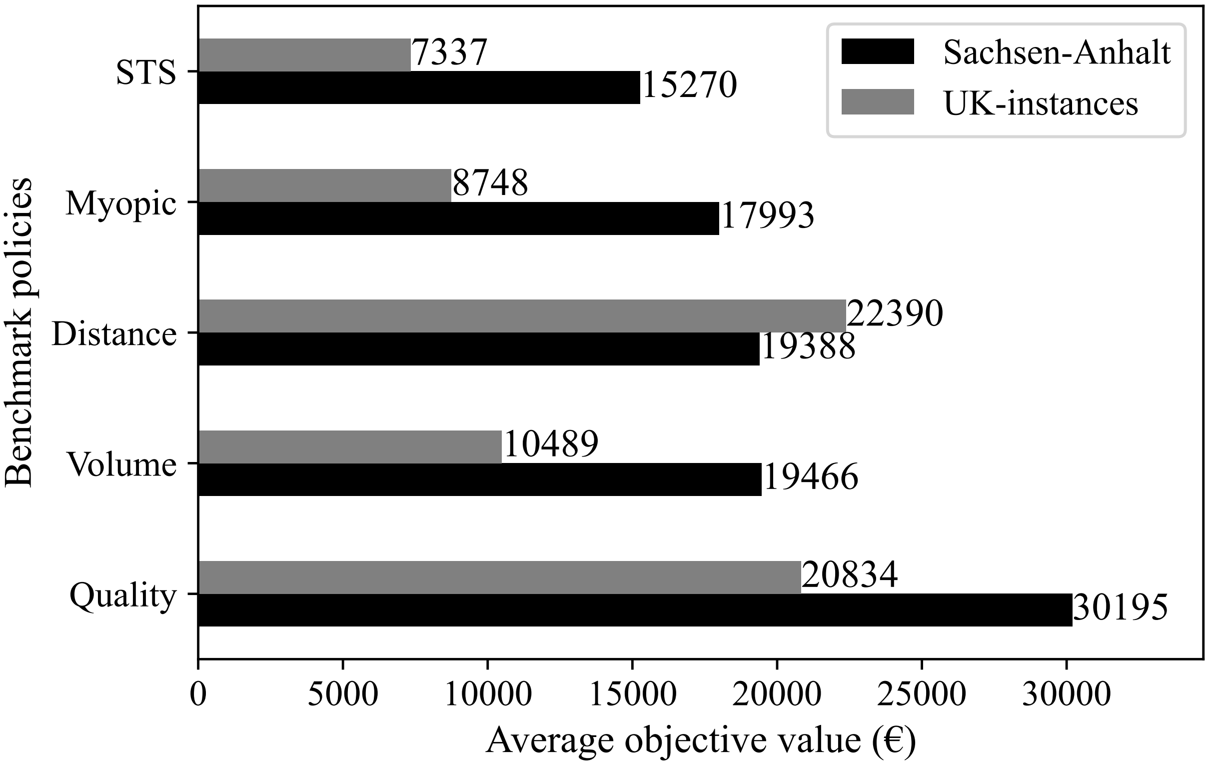


Figure 3: Comparison benchmark policies for intra-period anticipation

STS, significantly outperforms the one that only considers the expected value of loss (*Myopic*). These findings indicate that the explicit incorporation of loss uncertainty significantly improves efficiency, resulting in average savings of 17.83% compared to the *Myopic*-policy. We note that this improvement results partially from the explicit intra-period anticipation and partially from the implicit inter-period anticipation of *STS*. By considering uncertainty in loss, *STS* builds inventory to avoid costly backorders, as we show in Appendix A.3.

Inter- and Intra-Period anticipation. Next, we analyze the value of combining inter-period with intra-period anticipation. To this end, we compare the proposed policies for inter-period anticipation in Figure 4. We also depict the values for *Myopic* and *STS*. We observe that for both instances, our proposed policy *STM* performs significantly better than all the benchmark policies. Thus, an explicit intra- and inter-period anticipation is very valuable. We further observe that policies *STS* and *Myopic+M* perform comparably similar. The first focuses on intra-period anticipation, the latter on inter-period anticipation. This indicates that both parts are important when searching for effective decisions. However, the larger gap of the combination of *STS* and *Myopic+M*, *STS+M* and our policy *STM* illustrates, that ideally, an integrated anticipation of intra- and inter-periodical uncertainties should be chosen.

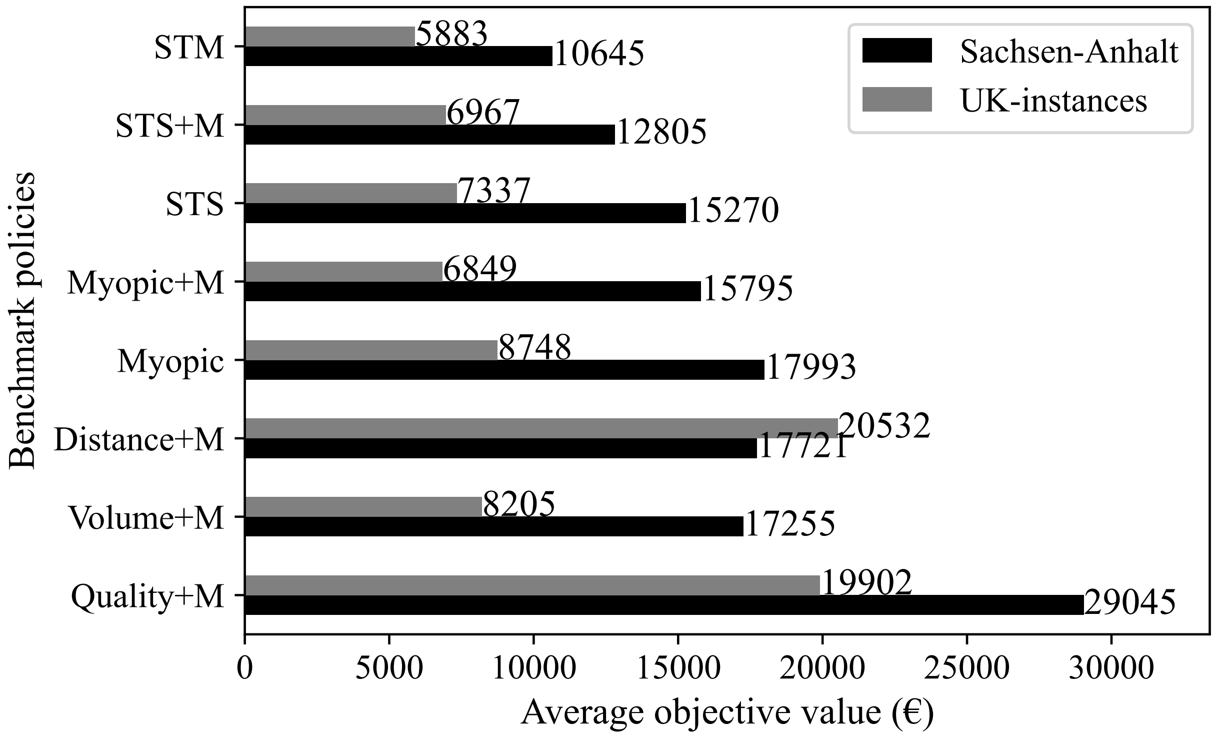


Figure 4: Comparison benchmark policies for intra- and inter-period anticipation.

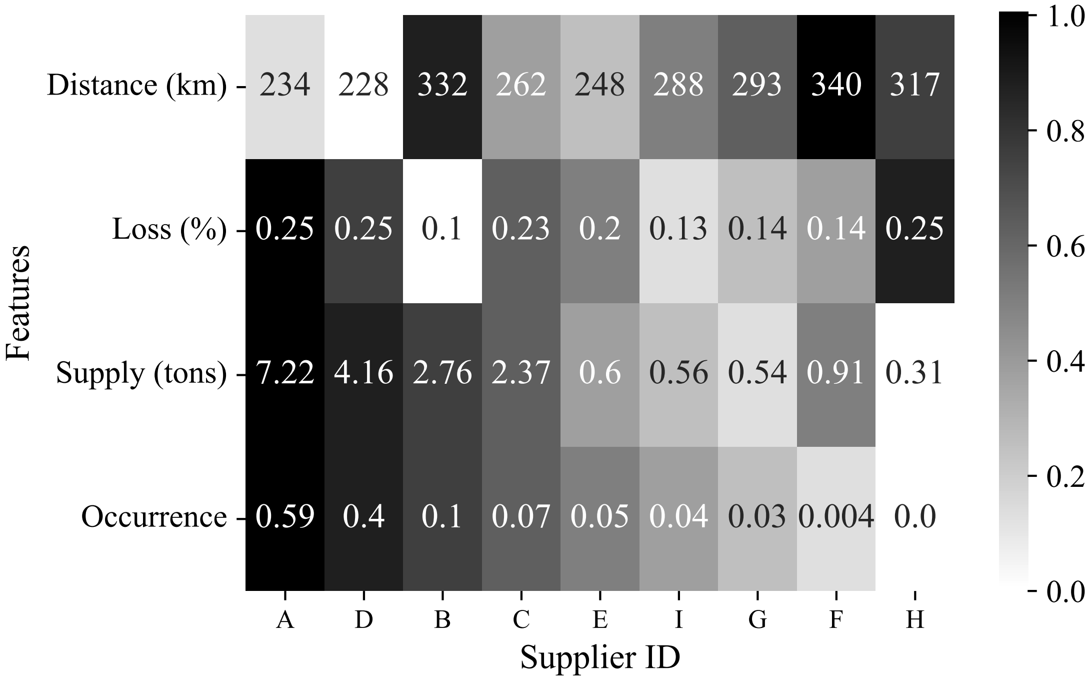
5.4. Analysis

Next, we analyse the decision making of our method and the general behavior of the problem.

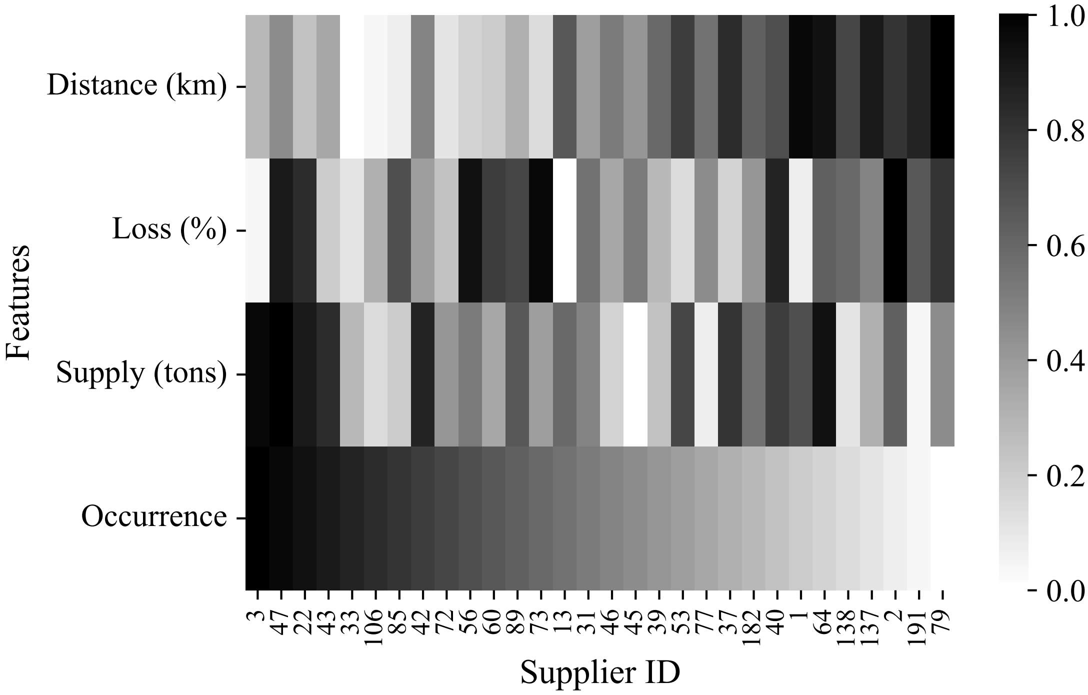
5.4.1. Decision making

First, we analyze the characteristics of the suppliers and their impact on the selection frequency and the general percentage of occurrences in the periods, defined as the percentage of periods a supplier is visited.

Figure 5a shows the occurrences for the nine suppliers (A-I) in Sachsen-Anhalt. It also shows the corresponding characteristics of the suppliers similar to the presented benchmark heuristics: the travel distance from the depot, the expected quality loss percentage, and the average increase in supply volume. The suppliers are sorted by occurrence values. The shadings of the individual cells indicate a supplier's value relative to the values of the other suppliers. Darker shades represent relatively large values while lighter shades represent comparably small values. We observe that the occurrence values depend on all three factors, distance, quality loss, and supply volume. The closest supplier is ranked second, the supplier with the highest quality third, and the supplier with the highest supply value is selected most frequently. In general, for the Sachsen-Anhalt case, the supply volume is the most important feature, followed by distance. Notably, the quality loss is not as relevant. The first two suppliers A and D have the highest expected loss, but since



(a) Sachsen-Anhalt case.



(b) UK-instances.

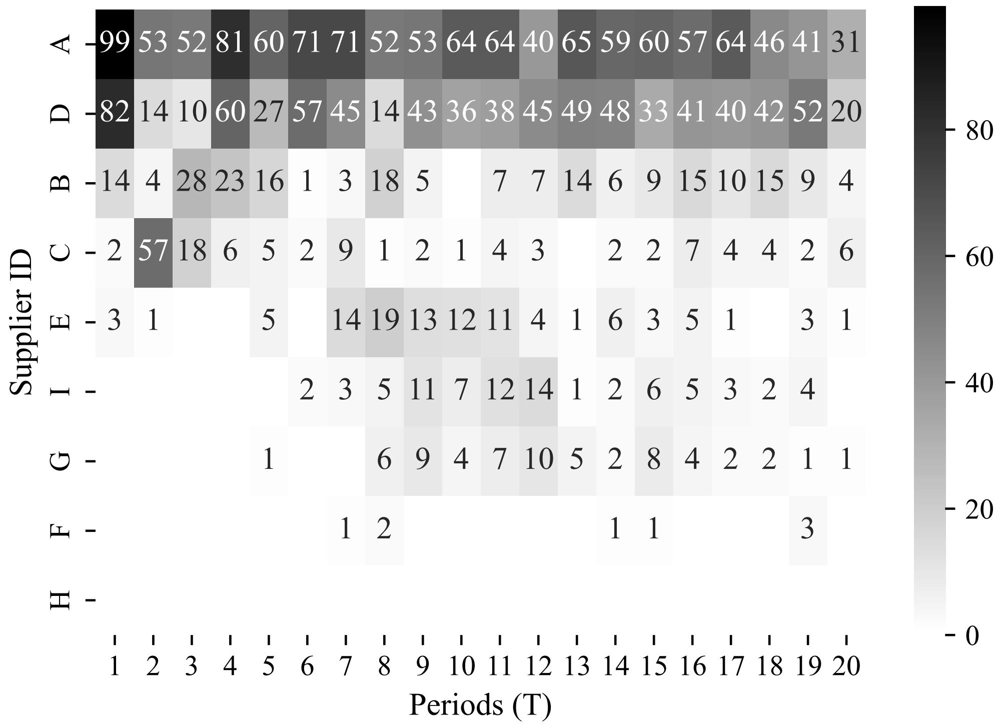
Figure 5: Analysis of the selection of suppliers based on their features

the overall supply volumes are large and the distances relatively short, they are most frequently selected anyway. However, for other suppliers, the selection criteria become more complex. When comparing suppliers B and C, we observe that while the offer volumes are comparably similar and the distance of C is smaller, supplier B is selected more frequently. This results from the interplay of loss, geography, and volume. First, supplier B has the smallest expected loss value. Second, more subtle, when visiting supplier B, supplier A is directly on the route. Given that the loss of supplier B is small, the spare room for additional supply is higher compared to supplier C.

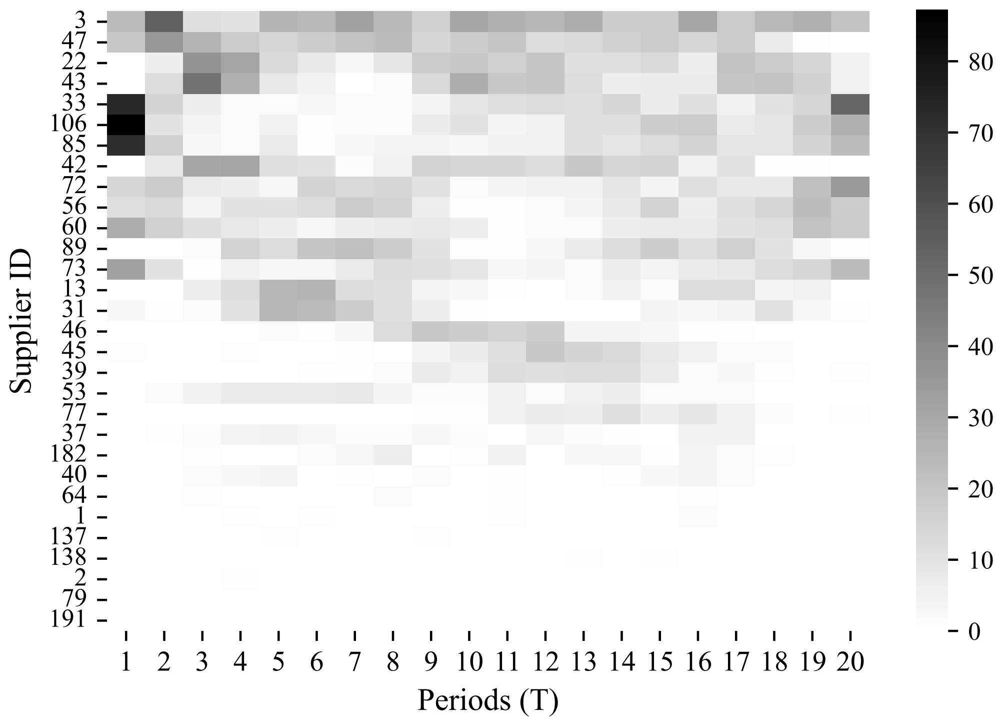
We show the results for the UK-instances in Figure 5b. Due to the larger number of suppliers, the depiction of individual values in the graph is omitted. Compared to the Sachsen-Anhalt case, we observe a similar pattern: Selection frequency depends on volume, distance, and expected loss. However, for this setting, the offer volume is slightly less important while the distance plays a more significant part in the selection of suppliers. Sorted by occurrences, the distance shading shows a relatively steady development from light to dark. For the supply, the first suppliers are again the ones with large supply volumes, however, there are several (further away) suppliers with larger supply amounts but less frequent occurrences (e.g., suppliers 37 and 64). One explanation for the different importance of the features could be that since there are more suppliers in the system, adjacent suppliers close to the depot can be visited more often together by one vehicle.

Next, we analyze at what times the suppliers are visited. To this end, for every period, we plot the percentage of times a supplier is visited. For the Sachsen-Anhalt case, the results are visualized in Figure 6a. The x-axis shows the periods, the y-axis the suppliers sorted by occurrences. We observe that suppliers A and D are visited very frequently. We further observe different consistent patterns for the remaining suppliers. For example, supplier C is visited mainly in the second period but supplier D rarely. At that time, supply of supplier D might be consumed and supplier C is visited instead. In contrast, visits to supplier B are more balanced over the periods. This supplier might be a general "backup" supplier and visited when supply of the two main suppliers A and D is limited. Interestingly, we observe some consistency for suppliers E, I, and G. They are mainly visited in the middle of the time horizon. For those suppliers, the increase in supply per period is small and the visits occur when they accumulated a "critical mass" worth collecting. Finally, we note that in the last period, the number of suppliers visited is comparably small. The inter-period anticipation of our policy collects sufficient supply in earlier periods to avoid sending out a vehicle in the last period in many cases.

The UK-results are shown in Figure 6b. Again, the suppliers are sorted by percentages of occurrence in the states. Interestingly, in the first period, different suppliers are selected than in later periods. Consulting Figure 5b, we observe that these suppliers are relatively close to the depot but have limited supply volumes. The visit of the suppliers with higher supply volumes is postponed to the second period. Here, our method anticipates the supply increase from period one to period two. Instead of sending a vehicle to the suppliers in the first period and returning to the depot half-empty, it satisfies the demand with nearby suppliers in the first period. In the second



(a) Sachsen-Anhalt case.



(b) UK-instances.

Figure 6: Supplier selection percentage over the planning horizon

period, suppliers 3 and 47 are visited individually, each consuming an entire vehicle capacity. As before, we can also observe a time-consistent pattern for many suppliers, e.g., for suppliers 13, 31, 46 and 45.

5.4.2. Problem dimensions

Next, we analyze the impact of changes in loss volatility and processing capacity. For the following analysis, we increase the number of instance realizations to 100 to allow for smoother values.

Loss volatility. One important feature in the considered problem is the uncertainty in quality loss. To further investigate the impact of loss uncertainty, we vary the volatility. Besides our original variation with a coefficient of variation (COV) of 0.1, COVs of 0, 0.2, and 0.5 are tested for the Sachsen-Anhalt case and policies *Myopic*, *STS*, and *STM*. We further apply a variant of *STM* where loss is integrated in the scenarios via expected values only, policy *STM(q)*. The results are shown in Figure 7. The x-axis depicts the COV, the y-axis the objective values of the different policies. We observe that with increasing loss volatility, the *Myopic* policy performs increasingly worse. This can be expected since this policy only operates on expected values. In contrast, policy *STS* explicitly considers loss volatility in the scenarios. Its performance does not only stay constant but even increases with increasing volatility. The reason is again that with different loss values in the scenarios, *STS* builds inventory. We further observe that policy *STM* is not effected much by increasing COVs. Thus, even when volatility is very uncertain, our proposed policy proves to be quite effective. Finally, we observe that the difference between *STM* and *STM(q)* is rather small, about 3% for the case of 0.5 COV.

Processing capacity. In our main experiments, we assume a processing capacity of $\beta = 2$ times the daily demand. Since the (expected) expected loss over all suppliers is 0.2 in our instances, this means that inventory of about 1.6 times the daily demand can be produced per period. We now analyze the impact of processing capacity by testing $\beta = 1, 1.5, 2, 2.5, 3$ times the daily demand. We apply policies *STM*, *STS*, and *Myopic* to the set of new instances.

The results for Sachsen-Anhalt are shown in Figure 8. The x-axis shows the maximum processing capacity. The y-axis shows the relative difference to *STM* in our main setting with capacity of $\beta = 2$. We observe that with more capacity, improvements are marginal. Thus, for our main setting, production capacity is not a bottleneck. Once capacity decreases, we see an increase in cost. The reasons for the cost are threefold. First, with loss uncertainty and reduced capacity, the likelihood of costly backorders increases. Second, the policies may decide to travel longer routes for collecting supply of higher quality, i.e., smaller expected loss. Third, with limited capacity, inventory cannot be built to hedge against supply uncertainty in future periods.

We now investigate the impact of the capacity constraint on decision making in more detail. To this end, we plot the average utilization of the processing capacity over the time horizon. The

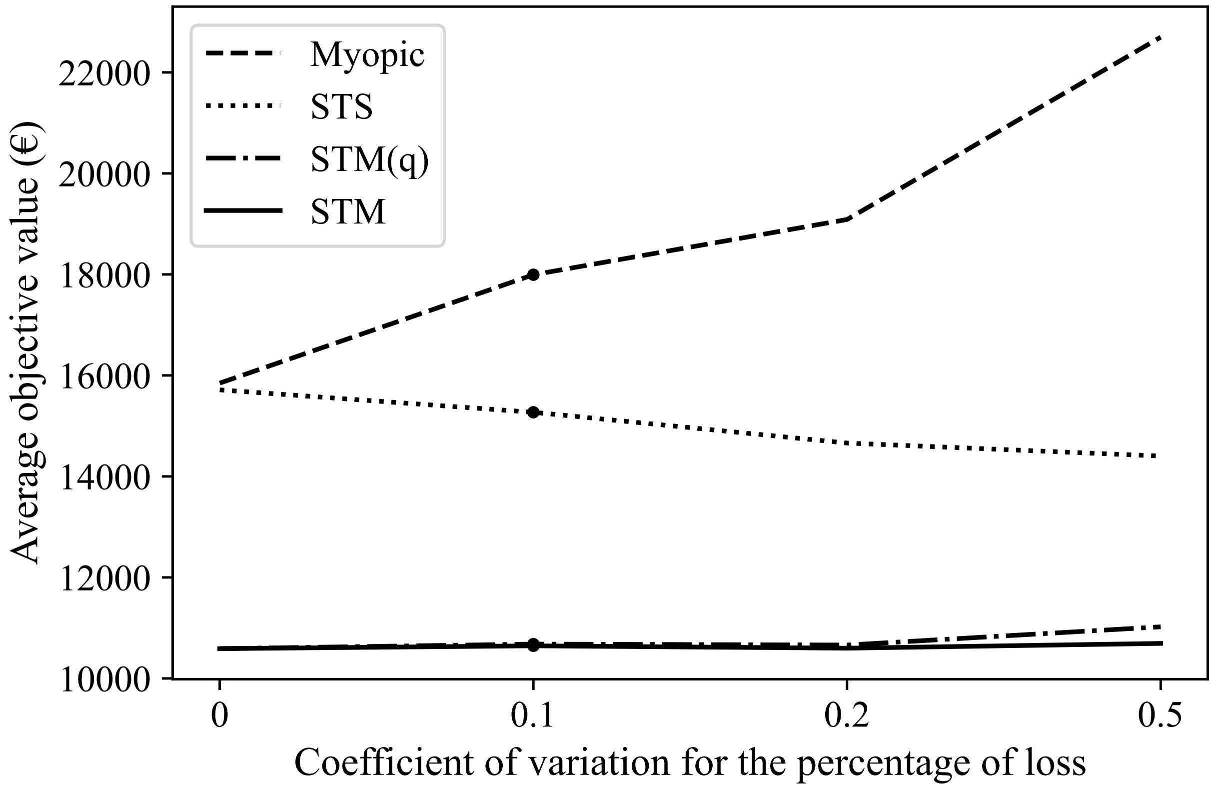


Figure 7: Changing the coefficient of variations of the loss distribution.

results are depicted in Figure 9. The x-axis shows the period, the y-axis the capacity utilization in tons (We recall that the net inventory demand is 10 tons per day). We make two main observations. First, with decreasing capacity limit, the usage becomes more levelled. This can be expected since with limited processing capacity, inventory cannot be build and every day, (nearly) all capacity is used to avoid costly backorders. However, we also observe some patterns in cases with high capacity limits. Given that the values are calculated over 100 runs, the differences cannot be explained by noise only. For example, we observe, that for higher capacities $\beta = 2$ and $\beta = 2.5$, in the first, fourth and seventh period, significantly more processing capacity is used while in other periods (e.g., two, five, and eight), less supply is processed. In the first period, no initial inventory is given and the policies decide to collect significantly more than needed to build an inventory for future periods. Consequently, the policy is flexible in the second period and may decide to save routing cost by collecting less. The same repeats over time, e.g., with periods 4 and 5, or with 16 and 17. These results indicate that there might be value in having more flexibility with respect to the processing capacity, e.g., by scheduling working shifts dynamically.

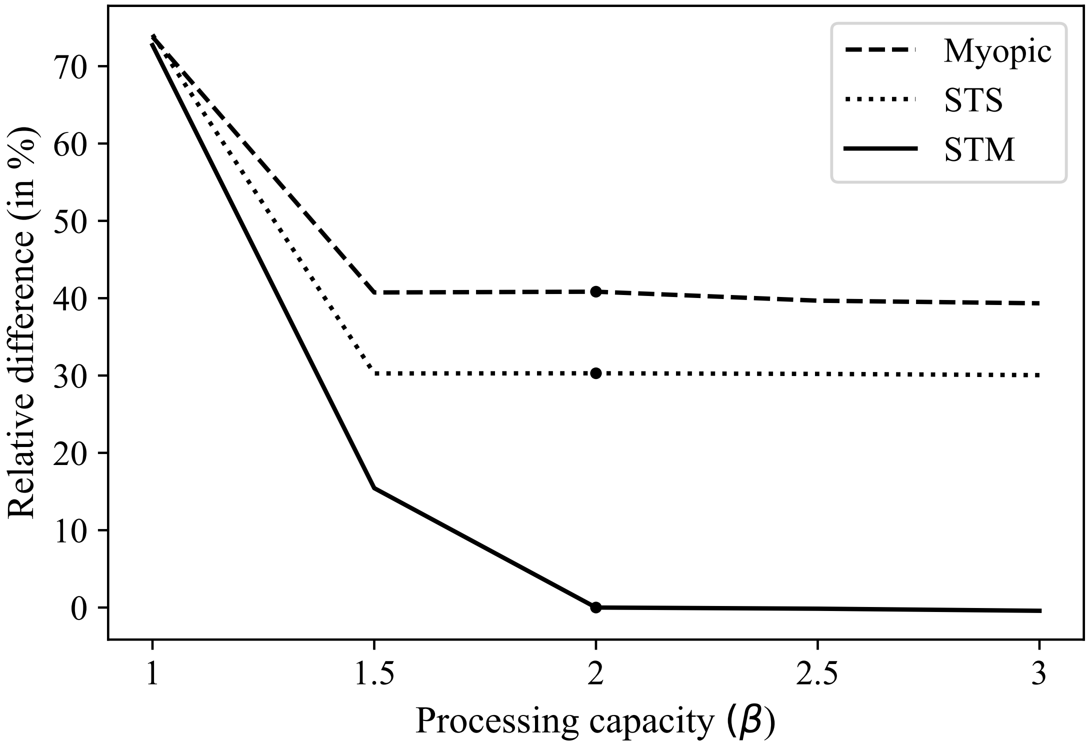


Figure 8: Performance for changing processing capacity parameter β (baseline $\beta = 2$).

6. Conclusion and outlook

In this paper, we have proposed effective and anticipatory policies for multi-period recycling collection operations with uncertainty in the available future supply as well as the supply's quality. There is a variety of future research opportunities. In our work, we have assumed that the quality loss probability distribution for each supplier is known and static. Future work may relax this assumption. For example, quality loss distributions may be unknown initially and could be approximated based on observed qualities via online learning. Here, future work may face a challenging trade-off between effective routing (exploitation) and effective learning (exploration). Another interesting avenue is a closer consideration of the processing of the supply. For example, instead of assuming inventory only for the processed inventory, future work may add another inventory for non-processed supply. Furthermore, in our work, we assumed processing capacity is a constraint. We further have shown that the capacity is not consumed equally per period but at distinct peak periods. Based on this insight, future research may add flexibility to processing capacity, e.g., allowing processing more material in one period but less in a subsequent period. Furthermore, processing capacity may itself be stochastic, either because of uncertain processing time, or, because of varying energy cost per period. Further, we observed consistency patterns in the periods specific suppliers are visited. Such consistency may allow for better planning and

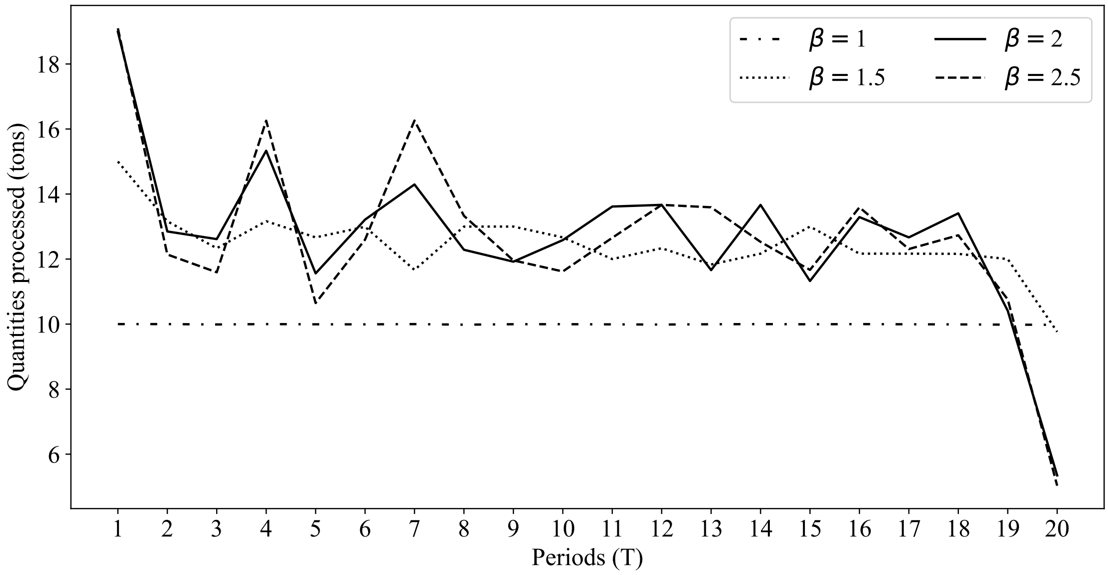


Figure 9: Processing capacity utilization over time for different capacity limits β.

cooperation with the waste collection facilities. Future research may investigate how consistency may be integrated in the decision making explicitly and to what cost.

Finally, our literature survey revealed work on routing problems with quality or yield uncertainty in rather limited. However, such uncertainty is present in a variety of problems, not only in recycling, but also in the collection of farming products or reverse logistics.

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Appendix

In the Appendix, we present an overview of the used notation and the results of additional experiments.

A.1. Literature details

In this section, we discuss the literature of Table 1 in detail.

A.1.1. VRP in the context of recycling collection

In the 1980s, the difficulties of routing municipal waste disposal are recognized by Raff (1983) as its own category of Vehicle Routing Problems (VRP). Its global impact and relevance to different societal dimensions, including resource management, energy utilization, ecological damages, and monetary costs, are acknowledged by Sushil (1990) and Bloemhof Ruwaard et al. (1995) early on. Hess et al. (2023) show that the optimization of recycle routing features various objectives, including the minimization of transportation and disposal costs, but also considering work hours, the length of collection routes, and potential environmental hazards. Govindan et al. (2015) address the potential reuse, recycling, and resource recovery of consumer, business, and industrial wastes, putting an emphasis on reverse logistics, enabling companies to include these wastes within their supply chain.

The optimization of vehicle routing within the context of recycling logistics has garnered significant attention in the literature. A comprehensive overview of the challenges and advancements in recycling logistics is presented in Hess et al. (2023) and Sar and Ghadimi (2023). These reviews emphasize the critical role of efficient routing in achieving sustainable waste management goals. They further highlight the necessity of tailored approaches to address the unique characteristics of recycling operations. Depending on the planning horizon, collection routing problems can be classified as single period or multi-period. Below, we discuss the studies related to these two categories.

For single-period collection problems, both deterministic and stochastic variants are explored. The deterministic version is addressed using exact and metaheuristic approaches (Kim et al., 2009; De Bruecker et al., 2018). Stochasticity often focuses on demand representing the quantity needed by customers or offered by suppliers. Various solution approaches are applied, including metaheuristics (Ismail and Loh, 2009; Marković et al., 2020), simheuristics (Gruler et al., 2017), dynamic programming (Cook and Lodree, 2017; Kyriakidis et al., 2020), chance constraints (Jammeli et al., 2021), and stochastic programming (Sasha Dong et al., 2022). Typically, uncertainty is revealed during customer visits, and demand samples are used for constructing routes.

Researchers have extended their focus to multi-period routing (MP-R) in recycling problems. Bogh et al. (2014) explores deterministic multi-period routing in a case study involving paper and glass recycling cubes. The authors propose a heuristic to suggest an alternative for the company's collection-based payment structures. In other studies, exact methods are applied to address multi-period routing with additional constraints such as due dates. Archetti et al. (2015) proposes a Branch & Cut approach, while Larrain et al. (2019) introduces a variable Mixed Integer Program neighborhood descent algorithm.

In addition, literature addresses the dynamic and stochastic aspects of multi-period collection operations. Typically, dynamism is addressed in the disclosure of customer orders and the subsequent updating of collection routes. As in the single-period scenario, demand stochasticity remains a prominent issue. Albareda-Sambola et al. (2014) introduce a rule-based method to determine whether a customer is visited based on the amount of supply revealed. Furthermore, researchers explore rolling horizon approaches as solution methods that solve period-by-period collection routes (Wen et al., 2010; Cordeau et al., 2015). Other methods, such as robust optimization, are employed for the construction of collection routes (Subramanyam et al., 2021).

Our work extends the research related to single-period waste collection problems. In contrast to previous work, our problem considers not only the supply but also the quality of the collected material as a source of uncertainty. Having uncertainty about the quality of the collected supply that can actually be used, generates a more volatile and dynamic environment, increasing the complexity of the problem. On the other hand, our study evaluates the implications of decisions not only on future routes but also on inventory levels and processing capacity utilization, which presents an additional dimension of complexity compared to multi-period routing problems.

A.1.2. Inventory routing

The Inventory Routing Problem (IRP) aims to find optimal inventory policies and vehicle routes to reduce supply chain costs. A comprehensive review of the IRP is presented in Andersson et al. (2010), Coelho et al. (2014), and Malladi and Sowlati (2018). Depending on the flow of products in the supply chain, inventory routing can be classified as either inbound routing (collection) or outbound routing (delivery), as mentioned in Cobb (2016). The research conducted in relation to these two categories and the multi-period characteristic is discussed below.

Inbound logistics, involving the collection of material from suppliers to a central facility, closely aligns with current research. Deterministic strategies for inbound logistics are explored, employing metaheuristics and hybrid approaches that combine heuristics with mixed-integer programs in studies such as Moin et al. (2011), Mjirda et al. (2014), Chitsaz et al. (2019), and Jieyu et al. (2024). On the other hand, exact methods, such as Branch & Cut approaches and the Benders decomposition algorithm, are developed in studies as Bertazzi et al. (2020), Chitsaz et al. (2020), and Chi and He (2023).

The literature on inbound logistics extends to stochastic scenarios, where variability in the supply of waste material from suppliers is a key focus. Mes et al. (2014) addresses the challenge of collecting waste from sensor-equipped underground containers, proposing a heuristic for supplier selection based on current supply levels. This heuristic is fine-tuned using optimal learning to adjust parameters dynamically. In contrast to the present work, their approach adopts a reactive strategy, adjusting decisions based on the available material in containers. Other approaches, including robust optimization and metaheuristics with scenarios, are explored in articles considering waste material collection (Liu et al., 2021; Frifita et al., 2022).

In the context of outbound logistics in recycling operations, the objective is to collect specific quantities of waste, with inventory decisions revolving around containers to minimize collection costs and prevent penalties from exceeding storage capacities. The primary source of uncertainty studied is the amount of waste in the containers. Nolz et al. (2014) presents a scenario-based method to address the collection of infectious medical waste. The proposed approaches are tested on a real case in France, demonstrating superior performance when considering uncertainty in collection decisions. Additionally, Markov et al. (2020) develop a metaheuristic with forecasting within a rolling horizon to tackle the collection problem, showing significant outperformance compared to deterministic policies. Lastly, in the energy context, Hasturk et al. (2024) introduce a cost-function approximation to solve a collection problem. Computational experiments validate the model across various scenarios, revealing substantial cost reductions compared to alternative solution methods.

Differing from the research on inventory routing, our study focuses on the influence of uncertainty on quality. This source of uncertainty affects inventory planning at the central facility to meet demand and determines the selection of suppliers to visit. The study is additionally constrained by processing capacity and dynamic changes between periods.

A.1.3. Uncertain supply and quality loss of materials in manufacturing and supply chains

Uncertainty in material quality and supply availability to generate new products is taken into account in manufacturing. The process of generating new products through the disassembly of collected materials, such as vehicles and electronic devices, is known as reverse materials requirements planning (RMRP). A comprehensive literature review of disassembly process-related problems is presented in Ilgin and Gupta (2010). In RMRP, the objective is to determine optimal disassembly schedules for used products, specifying the quantity of products and the timing of their disassembly. This must be done while satisfying the demand for individual parts and components within a given planning horizon without exceeding processing capacity. The most studied sources of uncertainty in this context are the uncertainty in demand and the quality of the collected products. Various approximate method approaches, including heuristics and metaheuristics, are explored in works such as Inderfurth and Langella (2006), Rickli and Camelio (2014), and Zhou et al. (2022). These approaches commonly employ techniques such as sampling from the sources of uncertainty or simulation to evaluate the quality of solutions. Additionally, approaches based on nonlinear models are also studied, as presented in Liu and Zhang (2018).

In the field of supply chain management (SCM), accounting for uncertainty factors is a critical imperative. The quality loss and supply of products, whether from suppliers or within closed-loop structures, constitute extensively studied sources of uncertainty that significantly impact the

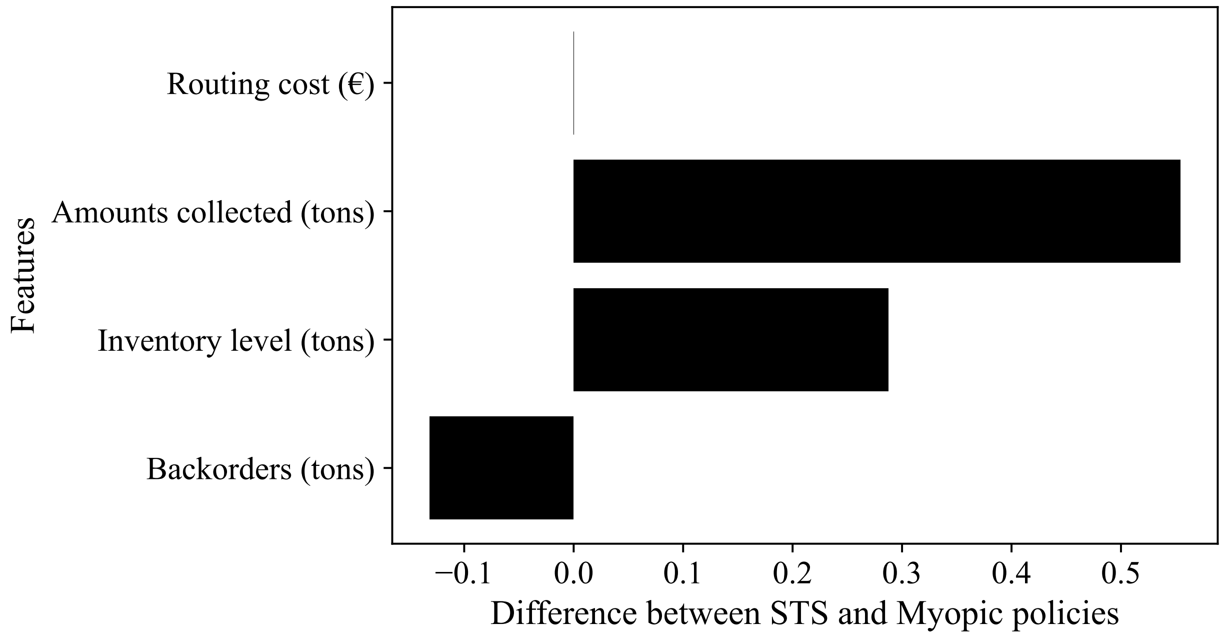

Figure A1: Comparison of decisions made by *STS* and *Myopic* policies in the single period version.

supply chain (Keyvanshokooh et al., 2016; Memişoğlu and Üster, 2021). To address the planning complexity while managing uncertainty, decomposition approaches like Benders and L-shaped algorithms gain widespread use (Biçe and Batun, 2021; Üster and Memişoğlu, 2018). Moreover, in addition to addressing uncertainty, these approaches also focus on robust designs, aiming to minimize the impact of variance in decision-making. For instance, in the context of biofuel production, efforts are directed towards robust designs, as demonstrated in the work by Li et al. (2023).

Our work differs from previous studies in two respects. First, the quality loss of the material collected in our study depends on supplier characteristics (location), directly influencing routing decisions and processing capacity utilization. This supplier-dependent quality has not been addressed in the literature, as quality loss by product type is assumed in most cases. Secondly, the problem addressed involves an additional layer of complexity, lacking a constant supply of material, which is also a source of uncertainty depending on the supplier.

A.2. Notation

Table A1 presents the notation used in the modeling of our problem and the solution methodology.

A.3. Difference between STS and Myopic

Figure A1 illustrates the differences in decision making between the *STS* and *Myopic* policies during the first period of the planning horizon, starting with an identical initial state for both

Notations	Definitions
Sets	
M	Set of suppliers
T	Set of periods
F	Set of vehicles
$\mathcal{V}$	Set of vertex, $\mathcal{V} := M \cup \{0\}$
$\mathcal{A}$	Set of arcs, $\mathcal{A} = \{(i,j) : i,j \in V \mid i \neq j\}$
$\mathcal{U}$	A set of nodes where $\mathcal{U} \subset \mathcal{V}$
$\delta^+(\mathcal{U})$	Set of arcs (i,j) with $i \in \mathcal{U}$ and $j \in \mathcal{V} \setminus \mathcal{U}$
$\delta^-(\mathcal{U})$	Set of arcs (i,j) with $j \in \mathcal{U}$ and $i \in \mathcal{V} \setminus \mathcal{U}$
T'	Set of periods in the lookahead horizon
Ω	Set of scenarios
Parameters	
d, β	Demand of new product and processing capacity at the depot
Q	Vehicle capacity
τ_{ij}	Travel time between i and j for $(i,j) \in \mathcal{V}$ with service time included
c	Cost for every time unit traveled
f	Backorder cost per unit
l^{max}	Maximum working time per vehicle and period
μ^r_m, σ^r_m	Mean and standard deviation of probability distribution for supply increase value for supplier $m \in M$
$\mu^\phi_m, \sigma^\phi_m$	Mean and standard deviation of probability distribution for quality loss percentage value for supplier $m \in M$
$r_{mt'\omega}, \phi_{mt'\omega}$	Realization of increase in supply and quality loss percentage for supplier $m \in M$ at period $t' \in T'$ in scenario $\omega \in \Omega$
I_t	Net inventory at the depot in period $t \in T$
q_{mt}	Amount of supply available at supplier $m \in M$ at period $t \in T$
h	Size of the lookahead horizon
$\hat{\tau}_m$	Direct trip cost from the depot to every supplier $m \in M$
γ	Discount parameter for routing cost estimation
Variables	
z_{mvt}	Amount of supply to collect from supplier $m \in M$ by vehicle $v \in F$ at period $t \in T$
x_{ijvt}	Equal to 1 if the arc $(i,j) \in \mathcal{A}$ is activated in the route of vehicle $v \in F$ at period $t \in T$, 0 otherwise
e_{mft}	Equal to 1 if supplier $m \in M$ is visited by vehicle $v \in F$ at period $t \in T$, 0 otherwise
$z'_{mt'\omega}$	Amount of supply to be collected from supplier $m \in M$ in period $t' \in T'$ in the scenario $\omega \in \Omega$
$e'_{mt'\omega}$	Equal to 1 if supplier $m \in M$ is visited at period $t' \in T'$ in the scenario $\omega \in \Omega$
$I'_{t'\omega}$	Inventory level of new product at the end of period $t' \in T'$ in the scenario $\omega \in \Omega$
$y_{t'\omega}$	Backorder amount variable at period $t' \in T'$ in scenario $\omega \in \Omega$
$k_{mt'\omega}$	Inventory level of waste material from supplier $m \in M$ at the end of period $t' \in T'$ in the scenario $\omega \in \Omega$

Table A1: Overview of notation

policies. The figure shows the difference in routing cost, supply collected, resulting inventory and backorders. The values are calculated as the difference between the average value for STS and the average value for *Myopic*.

We observe that the routing cost is the same for both policies. However, the STS policy collects a larger amount than the *Myopic* policy (0.5 tons more on average). This additional amount results from the different loss-scenarios and leads to a higher inventory and fewer backorders.

A.4. Vehicle parameters

In the following, we analyze the performance of the policies in case vehicle parameters change. First, we investigate the impact of the vehicle capacity by test capacities of 5 tons (small truck) and 20 tons (truck with trailer). We compare the changes of the policies with respect to STM and our basis Sachsen-Anhalt setting with 10 tons capacity. The results are shown in Figure A2. We observe that our policy becomes even more effective in case the capacity is increased. Notably, for the 20 tons instances, policy *Myopic* achieves similar results as our policy with 10 tons capacity.

In case of very limited truck capacities of 5 tons, we observe a substantial increase in cost. Thus, having sufficiently large trucks is very important for the company.

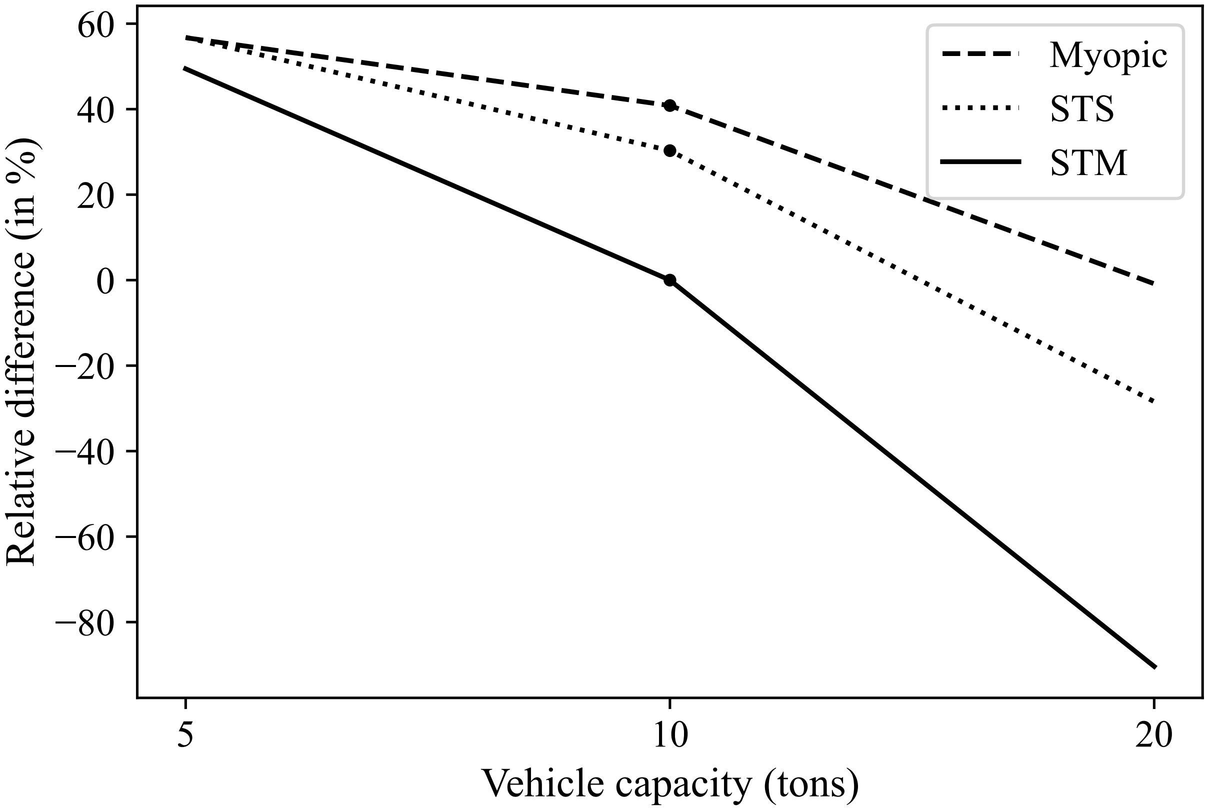

Figure A2: Changing of vehicle capacity

Next, we analyze the routing cost. In our main experiments, we assume cost of 1.5 Euros per minute travel. Now, we vary the cost from 0.5, 1.0 up to 3 Euros per minute. The results are shown in Figure A3, again relative to the base setting. We observe that regardless the routing cost, *STM* performs superior. We further observe that the policy is particularly effective when the routing cost is small.

A.5. Backorder cost

In this section, we show the results for varying backorder cost in Figure A4. We run the three policies for backorder cost of 50, 100, 250, 500, 1000, and 2000 per ton. We observe that the results remain comparably stable. Ideally, backorder cost is avoided. Only for cases with (unrealistically) cheap backorder cost of 50, all policies switch from collection routing to backordering.

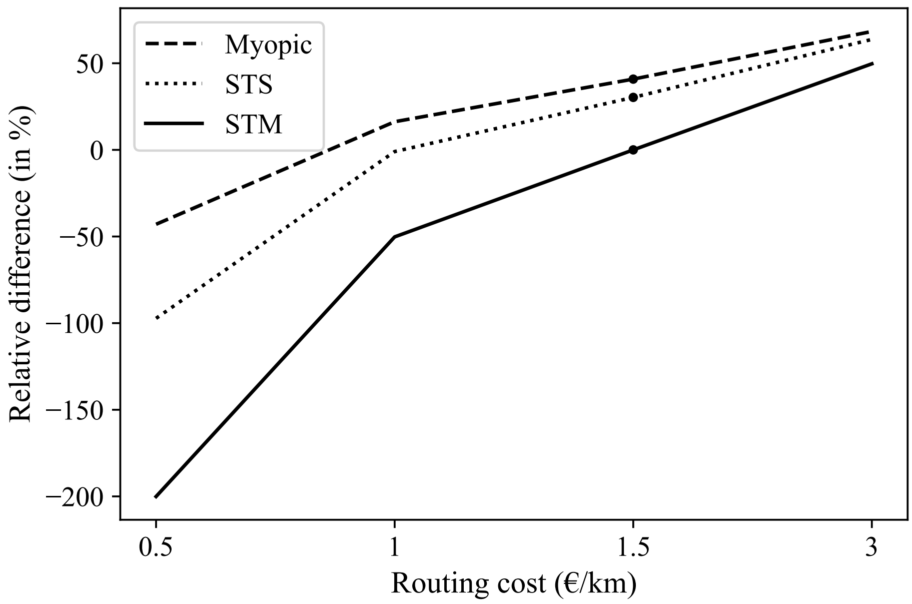

Figure A3: Changing of routing cost

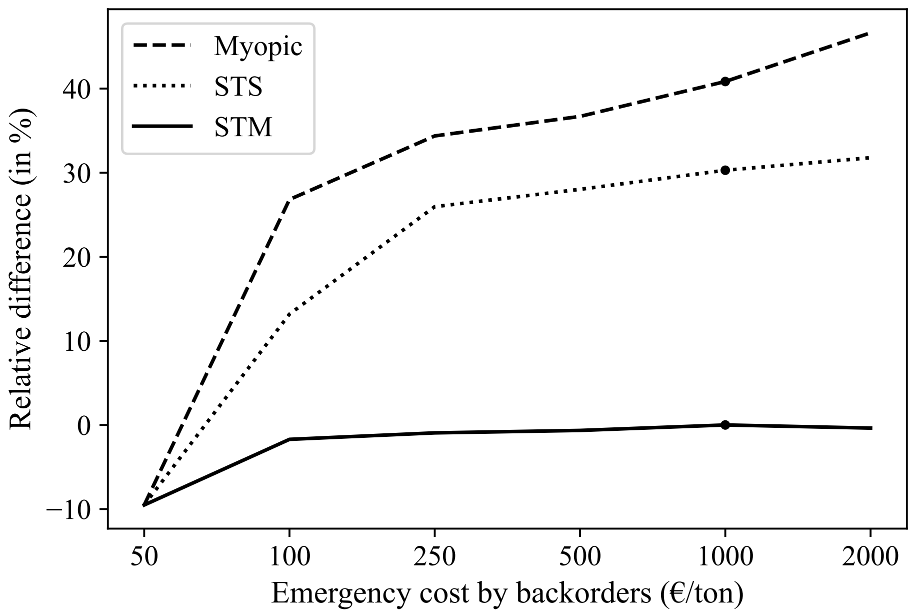

Figure A4: Changing of emergency cost by backorders

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